

# A Low-Complexity Algorithm for Worst-Case Utility Maximization in Multiuser MISO Downlink

Kun-Yu Wang<sup>\*</sup>, Haining Wang<sup>†</sup>, Zhi Ding<sup>†</sup>, and Chong-Yung Chi<sup>\*</sup>

<sup>\*</sup>Institute of Communications Engineering  
National Tsing Hua University,  
Hsinchu, Taiwan 30013

E-mail: kunyuwang7@gmail.com, cychi@ee.nthu.edu.tw

<sup>†</sup>Department of Electrical and Computer Engineering  
University of California, Davis,  
Davis, CA 95616

E-mail: {whnzinc, zding.ucdavis}@gmail.com

**Abstract**—This work considers worst-case utility maximization (WCUM) problem for a downlink wireless system where a multi-antenna base station communicates with multiple single-antenna users. Specifically, we jointly design transmit covariance matrices for each user to robustly maximize the worst-case (i.e., minimum) system utility function under channel estimation errors bounded within a spherical region. This problem has been shown to be NP-hard, and so any algorithms for finding the optimal solution may suffer from prohibitively high complexity. In view of this, we seek an efficient and more accurate suboptimal solution for the WCUM problem. A low-complexity iterative WCUM algorithm is proposed for this nonconvex problem by solving two convex problems alternatively. We also show the convergence of the proposed algorithm, and prove its Pareto optimality to the WCUM problem. Some simulation results are presented to demonstrate its substantial performance gain and higher computational efficiency over existing algorithms.

## I. INTRODUCTION

Linear transmit precoding has been recognized as an important technique for capacity improvement and low implementation complexity. Considering a transmit design for system utility maximization in a single-cell multiuser multiple-input single-output (MISO) wireless system, several works have focused on finding the optimal beamforming solution for the problem [1]–[3]. However, since the problem is NP-hard in general [4], the computation complexity of any algorithms for finding the optimal solution can be prohibitively high, rendering the convergence rate very slow. Therefore, those algorithms may be infeasible for real-time implementation. In light of this, considerable attention has been shifted towards finding a more accurate and computationally efficient solution for the system utility maximization problem. Assuming that the base station (BS) can perfectly acquire the channel state information (CSI) of the users, some efficient suboptimal algorithms have been proposed for the utility maximization problem [5]–[7]. The authors in [7] propose a suboptimal algorithm based on the idea of successive convex approximation (SCA) for a weighted sum rate maximization problem, with numerical results showing that their algorithm outperforms existing ones.

However, in practical situations, it is inevitable to have channel estimation errors at the BS due to finite training resource (e.g., power and signal length) or limited feedback bandwidth [8]. If one uses inaccurate CSI directly to design

the transmit precoders, then the system performance may be degraded seriously. In view of this, we design a transmit precoder by taking CSI errors into account. Also, in a slow fading channel, under a strict constraint on quality of service (QoS), the system must be designed for the worst-case scenario [9]. In this paper, the CSI errors are assumed be bounded spherically, and the transmit covariance matrices for all the users are designed to maximize the worst-case (i.e., minimum) system utility function against any possible CSI error, subject to a total transmit power constraint. However, the worst-case utility maximization (WCUM) problem is nonconvex and is generally hard to efficiently solve. Therefore, very few efficient algorithms have been reported for the WCUM problem.

One approach to developing a suboptimal solution for the WCUM problem is based on SCA. Although the SCA-based algorithm presented in [7] relies on the assumption that the BS has perfect CSI of the users, the SCA-based concept can be easily extended for handling the WCUM problem. Specifically, we can apply a conservative convex approximation to the WCUM problem, and then iteratively update the associated parameters to improve the performance. Since a conservative approximation is used in SCA-based algorithms, performance of such algorithm would be degraded. Hence in this paper, we propose a low-complexity algorithm for the WCUM problem without any conservative approximations. In the proposed WCUM algorithm, two convex optimization problems are solved alternatively. The proposed algorithm is guaranteed to converge, and the limit point is Pareto optimal to the WCUM problem. Some simulation results are presented to show that the proposed WCUM algorithm significantly outperforms the SCA-based algorithm, and the former has lower computational complexity than the later.

## II. SYSTEM MODEL AND PROBLEM STATEMENT

### A. System Model

We consider a single-cell multiuser multiple-input single-output (MISO) wireless system, where a BS equipped with  $N_t$  antennas communicates with  $K$  single-antenna users. The transmitted signal at the BS is given by

$$\mathbf{x}(t) = \sum_{k=1}^K \mathbf{x}_k(t), \quad (1)$$

where  $\mathbf{x}_k(t) \in \mathbb{C}^{N_t}$  denotes the information-bearing signal for the  $k$ th user. With (1), the received signal at the  $k$ th user can be represented as

$$\mathbf{y}_k(t) = \mathbf{h}_k^H \mathbf{x}(t) + n_k(t), \quad (2)$$

where  $\mathbf{h}_k \in \mathbb{C}^{N_t}$  denotes the channel vector between the BS and the  $k$ th user, and  $n_k(t)$  is the additive noise at the  $k$ th user with power  $\sigma_k^2 > 0$ .

Assuming that  $\mathbf{x}_k(t)$  is complex Gaussian distributed with zero mean and covariance matrix  $\mathbf{Q}_k \succeq \mathbf{0}$  (positive semidefinite (PSD) matrix), i.e.,  $\mathbf{x}_k(t) \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_k)$ , and considering single-user detection, the achievable rate of the  $k$ th user can be represented as (in bits/sec/Hz):

$$R_k(\{\mathbf{Q}_i\}_{i=1}^K, \mathbf{h}_k) = \log_2 \left( 1 + \frac{\mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k}{\sum_{\ell \neq k} \mathbf{h}_k^H \mathbf{Q}_\ell \mathbf{h}_k + \sigma_k^2} \right). \quad (3)$$

The goal of the transmit precoding is to design the transmit covariance matrices  $\{\mathbf{Q}_k\}_{k=1}^K$  such that a system utility function  $U(R_1, \dots, R_K)$  can be maximized, as formulated as the following optimization problem:

$$\max_{\substack{\mathbf{Q}_k \in \mathbb{H}^{N_t}, \\ k=1, \dots, K}} U(\{\mathbf{Q}_i(\{\mathbf{Q}_i\}_{i=1}^K, \mathbf{h}_k)\}_{k=1}^K) \quad (4a)$$

$$\text{s.t.} \quad \sum_{k=1}^K \text{Tr}(\mathbf{Q}_k) \leq P, \quad \mathbf{Q}_1, \dots, \mathbf{Q}_K \succeq \mathbf{0}, \quad (4b)$$

where  $\text{Tr}(\cdot)$  denotes the trace of a matrix, and  $P > 0$  is a preset maximum allowed total transmit power. We assume the system utility function  $U(R_1, \dots, R_K)$  to be strictly increasing and concave with respect to  $R_k$ , for  $k = 1, \dots, K$ , as satisfied by many practical system performance measures [4], e.g., sum-rate utility  $U(R_1, \dots, R_K) = (1/K) \sum_{k=1}^K R_k$ . Although problem (4) is not convex in general due to the nonconcave utility function with respect to  $\{\mathbf{Q}_k\}_{k=1}^K$ , it falls in the class of monotonic optimization problems [10], and the optimal transmit covariance matrices can be obtained [1]–[3], in spite of high computational complexity.

### B. Problem Statement

In practice, the BS cannot perfectly acquire the CSI from users due to finite training power or limited feedback bandwidth [8]. In this work, the true channel is modeled as

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k, \quad (5)$$

where  $\hat{\mathbf{h}}_k \in \mathbb{C}^{N_t}$  denotes the channel estimate of  $\mathbf{h}_k$  at the BS, and  $\mathbf{e}_k \in \mathbb{C}^{N_t}$  is the corresponding CSI error vector which is assumed to lie in a norm ball with radius  $r_k > 0$ , i.e.,  $\|\mathbf{e}_k\| \leq r_k$ , where  $\|\cdot\|$  denotes the vector 2-norm.

Our goal is to jointly design the transmit covariance matrices  $\{\mathbf{Q}_k\}_{k=1}^K$  such that the worst-case (i.e., minimum) system utility over the CSI errors is maximized, subject to a total transmit power constraint. Mathematically, the WCUM problem can be

formulated as

$$\max_{\substack{\mathbf{Q}_k \in \mathbb{H}^{N_t}, \\ k=1, \dots, K}} \min_{\substack{\|\mathbf{e}_k\| \leq r_k, \\ k=1, \dots, K}} U(\{\mathbf{Q}_i(\{\mathbf{Q}_i\}_{i=1}^K, \hat{\mathbf{h}}_k + \mathbf{e}_k)\}_{k=1}^K) \quad (6a)$$

$$\text{s.t.} \quad \sum_{k=1}^K \text{Tr}(\mathbf{Q}_k) \leq P, \quad \mathbf{Q}_1, \dots, \mathbf{Q}_K \succeq \mathbf{0}, \quad (6b)$$

Equivalently, problem (6) can be reformulated as

$$\begin{aligned} & \max_{\substack{\mathbf{Q}_k \in \mathbb{H}^{N_t}, t_k \in \mathbb{R}, \\ k=1, \dots, K}} U(t_1, \dots, t_K) \quad (7a) \\ & \text{s.t.} \quad \frac{(\hat{\mathbf{h}}_k + \mathbf{e}_k)^H \mathbf{Q}_k (\hat{\mathbf{h}}_k + \mathbf{e}_k)}{\sum_{\ell \neq k} (\hat{\mathbf{h}}_k + \mathbf{e}_k)^H \mathbf{Q}_\ell (\hat{\mathbf{h}}_k + \mathbf{e}_k) + \sigma_k^2} \geq t_k, \\ & \quad \forall \|\mathbf{e}_k\| \leq r_k, \quad k = 1, \dots, K, \quad (7b) \\ & \quad \sum_{k=1}^K \text{Tr}(\mathbf{Q}_k) \leq P, \quad \mathbf{Q}_1, \dots, \mathbf{Q}_K \succeq \mathbf{0}, \quad (7c) \end{aligned}$$

where we have made the change of variables

$$R_k(\{\mathbf{Q}_i\}_{i=1}^K, \hat{\mathbf{h}}_k + \mathbf{e}_k) = \log_2(1 + t_k) \quad (8)$$

in the utility function. Problem (6), or equivalently problem (7), provides a maximum lower bound on the system utility function over the CSI errors, given that  $\{\hat{\mathbf{h}}_k\}_{k=1}^K$  is known at the BS. However, problem (7) is hard to solve due to the infinitely many nonconvex constraints in (7b). Before presenting the algorithm for efficiently and effectively handling problem (7), let us present a simulation example to demonstrate the essentiality of the WCUM design.

**Example:** Consider sum-rate utility for problem (7) and naive-CSI-based design. For naive-CSI-based design, the transmit covariance matrices are obtained by solving the conventional perfect-CSI-based problem (4), where the channel estimates  $\{\hat{\mathbf{h}}_k\}_{k=1}^K$  are used as if they were true channels. For each realization of the channel estimates  $\{\hat{\mathbf{h}}_k\}_{k=1}^K$  generated with complex Gaussian distribution, the optimal transmit covariance matrices for problem (7) and naive-CSI-based designs are obtained by applying branch-reduce-and-bound (BRB) algorithm reported in [3]. With the obtained transmit covariance matrices and the presumed channels  $\{\hat{\mathbf{h}}_k\}_{k=1}^K$ , the worst-case sum rates were determined to be the minimum value of  $R_k$  computed by (3) over  $10^6$  true channels  $\{\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k\}_{k=1}^K$ , where the simulated CSI errors  $\{\mathbf{e}_k\}_{k=1}^K$  are randomly and independently generated satisfying  $\|\mathbf{e}_k\| \leq r$  for  $k = 1, \dots, K$ . Figure 1 displays some simulation results of the achieved worst-case sum rate versus  $r$  for  $N_t = K = 2$ ,  $P = 10$  dB, and  $\sigma_1^2 = \dots = \sigma_K^2 = 0.01$ , where each result was obtained by averaging over 100 realizations of the channel estimates  $\{\hat{\mathbf{h}}_k\}_{k=1}^K$ . From this figure, one can see that improper transmit covariance matrices can yield a very low sum rate in the worst case, especially for large CSI error radius. ■

Since problem (7) is NP-hard in general [4], the computational complexity of any algorithms for finding the optimal solution of problem (7) can be prohibitively high, and thus it is infeasible for real-time implementation [1]–[3]. Next,

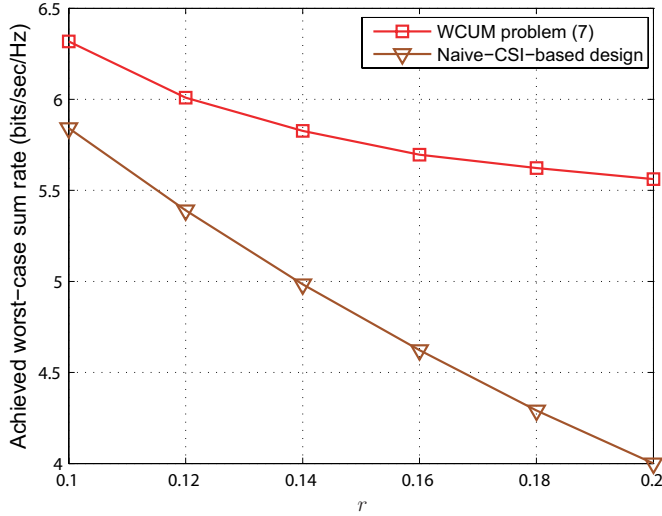


Fig. 1: Achieved worst-case sum rate vs. CSI error radius  $r$ .

we concentrate on the proposed low-complexity algorithm for finding a more accurate suboptimal solution for problem (7).

### III. A LOW-COMPLEXITY WCUM ALGORITHM

In this section, we first present the proposed iterative algorithm for problem (7), and then prove its convergence.

#### A. Proposed Algorithm

To proceed, let us apply S-lemma [11] to constraint (7b), and then problem (7) can be equivalently reformulated as (see, e.g., [3] for details):

$$\max_{\substack{\mathbf{Q}_k \in \mathbb{H}^{N_t}, \\ t_k, \lambda_k \in \mathbb{R}, \\ k=1, \dots, K}} U(t_1, \dots, t_K) \quad (9a)$$

$$\text{s.t. } \Phi_k(t_k, \lambda_k, \{\mathbf{Q}_i\}_{i=1}^K) \succeq \mathbf{0}, \quad k = 1, \dots, K, \quad (9b)$$

$$\sum_{k=1}^K \text{Tr}(\mathbf{Q}_k) \leq P, \quad (9c)$$

$$\lambda_k \geq 0, \quad \mathbf{Q}_k \succeq \mathbf{0}, \quad k = 1, \dots, K, \quad (9d)$$

where  $\lambda_1, \dots, \lambda_K \in \mathbb{R}$  are the introduced slack variables, and

$$\begin{aligned} \Phi_k(t_k, \lambda_k, \{\mathbf{Q}_i\}_{i=1}^K) \triangleq & \begin{bmatrix} \mathbf{I}_{N_t} \\ \hat{\mathbf{h}}_k^H \end{bmatrix} \left( \mathbf{Q}_k - t_k \sum_{\ell \neq k} \mathbf{Q}_\ell \right) \begin{bmatrix} \mathbf{I}_{N_t} \\ \hat{\mathbf{h}}_k^H \end{bmatrix}^H \\ & + \begin{bmatrix} \lambda_k \mathbf{I}_{N_t} & \mathbf{0} \\ \mathbf{0} & -\lambda_k r_k^2 - t_k \sigma_k^2 \end{bmatrix}, \quad k = 1, \dots, K. \end{aligned} \quad (10)$$

Although problem (9) is still not convex due to nonconvex constraint (9b), the problem has a more tractable form than problem (7). To develop the proposed algorithm, we need the following lemma.

**Lemma 1** Each eigenvalue of the matrix  $\Phi_k(t_k, \lambda_k, \{\mathbf{Q}_i\}_{i=1}^K)$  defined in (10) decreases with  $t_k$ , for  $k = 1, \dots, K$ .

*Proof:* By letting  $\delta_{k,j} \in \mathbb{R}$ ,  $j = 1, \dots, N_t + 1$ , be the eigenvalues of the matrix  $\Phi_k(t_k, \lambda_k, \{\mathbf{Q}_i\}_{i=1}^K)$  and denoting

$\mathbf{v}_{k,j} \in \mathbb{C}^{N_t+1}$  as the associated unit-norm eigenvector, we have

$$\begin{aligned} \delta_{k,j} &= \mathbf{v}_{k,j}^H \Phi_k(t_k, \lambda_k, \{\mathbf{Q}_i\}_{i=1}^K) \mathbf{v}_{k,j} \\ &= \mathbf{v}_{k,j}^H \mathbf{Y}_{k,k} \mathbf{v}_{k,j} + \lambda_k \mathbf{v}_{k,j}^H \begin{bmatrix} \mathbf{I}_{N_t} & \mathbf{0} \\ \mathbf{0} & -r_k^2 \end{bmatrix} \mathbf{v}_{k,j} \\ &\quad - t_k \left( \sum_{\ell \neq k} \mathbf{v}_{k,j}^H \mathbf{Y}_{k,\ell} \mathbf{v}_{k,j} + \sigma_k^2 \|\mathbf{v}_{k,j}\|_{N_t+1}^2 \right), \end{aligned} \quad (11)$$

where  $[\mathbf{a}]_j$  denotes  $j$ th entry of a vector  $\mathbf{a}$ , and

$$\mathbf{Y}_{k,\ell} \triangleq \begin{bmatrix} \mathbf{I}_{N_t} \\ \hat{\mathbf{h}}_k^H \end{bmatrix} \mathbf{Q}_\ell \begin{bmatrix} \mathbf{I}_{N_t} \\ \hat{\mathbf{h}}_k^H \end{bmatrix}^H \succeq \mathbf{0}, \quad \ell = 1, \dots, K. \quad (12)$$

Since  $\mathbf{Y}_{k,\ell}$  is a PSD matrix, from (11), one can easily show that the eigenvalue  $\delta_{k,j}$  decreases with  $t_k$ , for  $j = 1, \dots, N_t + 1$ . This proof is thus complete. ■

Problem (9) aims to maximize  $t_k$  (because the utility function is strictly increasing in  $t_k$ ). According to Lemma 1, one can maximize  $t_k$  by maximizing the minimum eigenvalue of the matrix  $\Phi_k(t_k, \lambda_k, \{\mathbf{Q}_i\}_{i=1}^K)$  in (9b) (which only involves  $t_k$ ). Also, it can be inferred that the minimum eigenvalue of  $\Phi_k(t_k, \lambda_k, \{\mathbf{Q}_i\}_{i=1}^K)$  must be zero as the optimal solution of problem (9) is achieved. Based on these facts, let us consider the following iterative approach for dealing with problem (9). At the  $m$ th iteration,  $t_1, \dots, t_K$  are updated by solving

$$\max_{\substack{t_k \in \mathbb{R}, \\ k=1, \dots, K}} U(t_1, \dots, t_K) \quad (13a)$$

$$\text{s.t. } \Phi_k(t_k, \lambda_k^{(m)}, \{\mathbf{Q}_i^{(m)}\}_{i=1}^K) \succeq \mathbf{0}, \quad \forall k. \quad (13b)$$

Let us denote the obtained optimal utility value in problem (13) as  $\tilde{U}^{(m)}(\{\mathbf{Q}_k^{(m)}, \lambda_k^{(m)}\}_{k=1}^K)$ , in which  $\mathbf{Q}_k^{(m)}$  and  $\lambda_k^{(m)}$ ,  $k = 1, \dots, K$ , are obtained by solving

$$\max_{\substack{\mathbf{Q}_k \in \mathbb{H}^{N_t}, \\ \lambda_k, z_k \in \mathbb{R}, \\ k=1, \dots, K}} \Psi \triangleq \sum_{k=1}^K z_k \quad (14a)$$

$$\text{s.t. } \Phi_k(t_k^{(m-1)}, \lambda_k, \{\mathbf{Q}_i\}_{i=1}^K) - z_k \mathbf{I}_{N_t+1} \succeq \mathbf{0}, \quad \forall k, \quad (14b)$$

$$\sum_{k=1}^K \text{Tr}(\mathbf{Q}_k) \leq P, \quad (14c)$$

$$\lambda_k \geq 0, \quad \mathbf{Q}_k \succeq \mathbf{0}, \quad z_k \geq 0, \quad k = 1, \dots, K. \quad (14d)$$

The obtained optimal value of  $\Psi$  in problem (14) is denoted as  $\Psi^{(m)}(\{t_k^{(m-1)}\}_{k=1}^K)$ , in which  $t_k^{(m-1)}$ ,  $k = 1, \dots, K$ , represent the solution obtained by solving problem (13) from the  $(m-1)$ th iteration. Problems (13) and (14) are convex, and thus can be efficiently solved. Specifically, since the design variables  $t_1, \dots, t_K$  in the objective function and in the constraints of problem (13) are decoupled, and the objective function is strictly increasing in  $t_k$ , for  $k = 1, \dots, K$ , by Lemma 1, the optimal  $\{t_k\}_{k=1}^K$  can be separately obtained by simple bisection search; i.e., find a  $t_k$  such that the minimum eigenvalue of  $\Phi_k(t_k, \lambda_k^{(m)}, \{\mathbf{Q}_i^{(m)}\}_{i=1}^K)$  in (13b) is equal to zero. The obtained WCUM algorithm for problem (7), or equivalently problem (9), is summarized in Algorithm 1.

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**Algorithm 1** Proposed WCUM algorithm for problem (7).

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- 1: Obtain a feasible point  $\{t_k^{(0)}\}_{k=1}^K$  according to (17); set a solution accuracy  $\epsilon > 0$ ; and set iteration index  $m = 0$ .
  - 2: **repeat**
  - 3:   Update  $m := m + 1$ .
  - 4:   Obtain  $\{\mathbf{Q}_k^{(m)}, \lambda_k^{(m)}\}_{k=1}^K$  by solving problem (14).
  - 5:   Obtain  $\{t_k^{(m)}\}_{k=1}^K$  by solving problem (13).
  - 6: **until** the predefined stopping criterion is met, e.g.,  $|\tilde{U}^{(m)} - \tilde{U}^{(m-1)}| \leq \epsilon$ .
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An initial feasible point  $\{t_k^{(0)}\}_{k=1}^K$  for solving problem (14) in the first iteration can be obtained by finding a feasible point of problem (7). Let the transmit covariance matrices be of rank one, i.e.,  $\mathbf{Q}_k = \mathbf{w}_k \mathbf{w}_k^H$  for  $k = 1, \dots, K$ , where  $\mathbf{w}_k \in \mathbb{C}^{N_t}$ ,  $k = 1, \dots, K$ , can be any arbitrary vectors such that the power constraint in (7c) is satisfied, i.e.,  $\sum_{k=1}^K \|\mathbf{w}_k\|^2 \leq P$ . Therefore, from (7b), a feasible point  $\{t_k^{(0)}\}_{k=1}^K$  is given by

$$t_k^{(0)} = \frac{\min_{\|\mathbf{e}_k\| \leq r_k} |(\hat{\mathbf{h}}_k + \mathbf{e}_k)^H \mathbf{w}_k|^2}{\max_{\|\mathbf{e}_k\| \leq r_k} \sum_{\ell \neq k} |(\hat{\mathbf{h}}_k + \mathbf{e}_k)^H \mathbf{w}_\ell|^2 + \sigma_k^2} = \frac{([\hat{\mathbf{h}}_k^H \mathbf{w}_k] - r_k \|\mathbf{w}_k\|)^2}{\sum_{\ell \neq k} (|\hat{\mathbf{h}}_k^H \mathbf{w}_\ell| + r_k \|\mathbf{w}_\ell\|)^2 + \sigma_k^2}, \quad k = 1, \dots, K, \quad (15)$$

where  $[a]^+ \triangleq \max\{a, 0\}$ . Letting

$$\mathbf{w}_k = \sqrt{\frac{P}{K}} \frac{\hat{\mathbf{h}}_k}{\|\hat{\mathbf{h}}_k\|}, \quad k = 1, \dots, K, \quad (16)$$

in (15) gives rise to

$$t_k^{(0)} = \frac{\frac{P}{K} ([\|\hat{\mathbf{h}}_k\| - r_k]^+)^2}{\sum_{\ell \neq k} \frac{P}{K} \left( \frac{|\hat{\mathbf{h}}_k^H \hat{\mathbf{h}}_\ell|}{\|\hat{\mathbf{h}}_\ell\|} + r_k \right)^2 + \sigma_k^2}, \quad k = 1, \dots, K. \quad (17)$$

### B. Convergence of Algorithm 1

In the following, we will show the convergence of Algorithm 1 and the limit point to be Pareto optimal to problem (7). To this end, we need the following lemma:

**Lemma 2** *If the objective value of problem (14) obtained in the  $m$ th iteration is positive, i.e.,  $\Psi^{(m)} > 0$ , then the system utility value can be increased by solving problem (13), i.e.,  $\tilde{U}^{(m)} > \tilde{U}^{(m-1)}$ , and the limit values  $z_1^*, \dots, z_K^*$  obtained by Algorithm 1 must be zero.*

*Proof:* Since  $\Psi^{(m)} = \sum_{k=1}^K z_k^{(m)}$  is positive, let us assume  $z_k^{(m)} > 0$  for some  $k$  without loss of generality. Therefore, from (14b), we have

$$\Phi_k(t_k^{(m-1)}, \lambda_k^{(m)}, \{\mathbf{Q}_i^{(m)}\}_{i=1}^K) \succ \mathbf{0}. \quad (18)$$

According to Lemma 1, we can always find a value, say  $t_k^{(m)}$ , such that  $t_k^{(m)} > t_k^{(m-1)}$  is feasible to problem (13). As a result, we have  $\tilde{U}^{(m)} > \tilde{U}^{(m-1)}$  since the objective function of problem (13) is strictly increasing in  $t_k$ .

Next, let us show that as Algorithm 1 converges, the limit values  $z_1^*, \dots, z_K^*$  are all zero by contradiction. If  $z_k^* \neq 0$ , then we can further increase the objective function of problem (13), which contradicts the premise that the algorithm has converged. This proof is thus complete. ■

Since  $\tilde{U}^{(m)}$  in Algorithm 1 is monotonically increasing in the iteration number  $m$  [by Lemma 2] and its value is bounded above due to finite total transmit power  $P$ , we can conclude that Algorithm 1 must converge. In the following proposition, the Pareto optimality of Algorithm 1 to problem (7) is established.

**Proposition 1** *The limit point  $\{\mathbf{Q}_k^*, t_k^*, \lambda_k^*\}_{k=1}^K$  obtained by Algorithm 1 is Pareto optimal to problem (7).*

*Proof:* To show that the limit point  $\{\mathbf{Q}_k^*, t_k^*, \lambda_k^*\}_{k=1}^K$  is Pareto optimal to problem (7), or equivalently problem (9), we first show that the limit point is a feasible point of problem (9). Since the point  $\{\mathbf{Q}_k^*, \lambda_k^*, z_k^*, t_k^*\}_{k=1}^K$  is feasible to problems (13) and (14), we have

$$\Phi_k(t_k^*, \lambda_k^*, \{\mathbf{Q}_i^*\}_{i=1}^K) \succeq \mathbf{0}, \quad k = 1, \dots, K, \quad (19a)$$

$$\sum_{k=1}^K \text{Tr}(\mathbf{Q}_k^*) \leq P, \quad (19b)$$

$$\lambda_k^* \geq 0, \quad \mathbf{Q}_k^* \succeq \mathbf{0}, \quad k = 1, \dots, K, \quad (19c)$$

where we use the fact  $z_1^* = \dots = z_K^* = 0$  in (19) according to Lemma 2. Comparing (19) with the feasible set of problem (9), one can conclude that the limit point  $\{\mathbf{Q}_k^*, \lambda_k^*, t_k^*\}_{k=1}^K$  generated by Algorithm 1 is feasible to problem (9).

Now, let us show that the limit point  $\{\mathbf{Q}_k^*, t_k^*, \lambda_k^*\}_{k=1}^K$  is Pareto optimal to problem (9) by contradiction. Suppose that the limit point is not Pareto optimal to problem (9). Then, according to the definition of Pareto optimality in [10], there exists a feasible solution to problem (9) such that  $U(\tilde{t}_1, \dots, \tilde{t}_K) > U(t_1^*, \dots, t_K^*)$ , where  $\tilde{t}_k > t_k^*$  and  $\tilde{t}_\ell \geq t_\ell^*$  for  $\ell \neq k$ . That is to say, the following constraint set is feasible [by Lemma 1]:

$$\Phi_k(t_k^*, \lambda_k, \{\mathbf{Q}_i\}_{i=1}^K) \succ \mathbf{0}, \quad (20a)$$

$$\Phi_\ell(t_\ell^*, \lambda_\ell, \{\mathbf{Q}_i\}_{i=1}^K) \succeq \mathbf{0}, \quad \forall \ell \neq k, \quad (20b)$$

$$\sum_{k=1}^K \text{Tr}(\mathbf{Q}_k) \leq P, \quad (20c)$$

$$\lambda_k \geq 0, \quad \mathbf{Q}_k \succeq \mathbf{0}, \quad k = 1, \dots, K. \quad (20d)$$

The feasibility of constraint (20) implies that there exists a solution  $\{\lambda_i, \mathbf{Q}_i\}_{i=1}^K$  such that the minimum eigenvalue of the matrix  $\Phi_k(t_k^*, \lambda_k, \{\mathbf{Q}_i\}_{i=1}^K)$  is positive, rendering  $z_k^* > 0$  in problem (14), which contradicts with the fact of  $z_k^* = 0$  by Lemma 2. Therefore, we have completed the proof that the limit point yielded by Algorithm 1 is Pareto optimal to problem (7). ■

## IV. SIMULATION RESULTS

We consider the wireless system as described in Section II with  $N_t = 4$  transmit antennas at the BS and  $K = 2$  single-antenna users, the total transmit power  $P = 10$  dB, and the



users' noise powers  $\sigma_1^2 = \sigma_2^2 = 0.01$ . For simplicity, the CSI error radii of all users are assumed to be identical, i.e.,  $r_1 = \dots = r_K \triangleq r$ . In each simulation trial, the presumed channels  $\{\hat{\mathbf{h}}_k\}_{k=1}^K$  are randomly and independently generated according to the standard complex Gaussian distribution.

Considering sum-rate utility for problem (7), we compare the worst-case sum rate performances of the proposed WCUM algorithm (Algorithm 1), SCA-based algorithm [7] and the optimal (i.e., maximum) worst-case sum rate obtained by BRB algorithm (which is a brute force approach) reported in [3]. The solution accuracy for Algorithm 1 and SCA-based algorithm is set to  $10^{-3}$ , i.e.,  $\epsilon = 10^{-3}$ , and the gap tolerance between the upper and lower bounds for the BRB algorithm is set to 0.1 as in [3]. The involved convex problems in the algorithms under test are solved using CVX [12]. Figure 2 shows the average sum rate versus CSI error radius  $r$ , where the sum rates obtained from the three algorithms are averaged over 100 realizations of the presumed channels  $\{\hat{\mathbf{h}}_k\}_{k=1}^K$ . From this figure, one can see that the proposed algorithm performs much better than SCA-based algorithm, with the performance gap about 1.8 bits/sec/Hz, but worse than BRB algorithm, with the performance gap between 0.8 bits/sec/Hz for  $r = 0.1$  and 0.2 bits/sec/Hz for  $r = 0.2$ . Note that the sum rate yielded by the proposed algorithm is closer to the optimal (i.e., maximum) sum rate as the error bound  $r$  grows, and that the performance gap between SCA-based algorithm and BRB algorithm is as high as 2.6 bits/sec/Hz for  $r = 0.1$  and 2 bits/sec/Hz for  $r = 0.2$ .

By our simulation experiences, we found that the proposed algorithm converges much faster than BRB algorithm and is more computationally efficient than SCA-based algorithm. As an illustration, the average computation times of BRB algorithm, SCA-based algorithm, and the proposed algorithm for obtaining the results shown in Fig. 2 for  $r = 0.1$  using a desktop PC with 3GHz CPU and 8GB RAM, are 550.9 secs, 123.9 secs, and 29.6 secs, respectively. The fast convergence rate of the proposed algorithm is due to the use of simple bisection method for solving problem (13), and due to the smaller size of problem (14) compared with those problems involved in BRB and SCA-based algorithms. Let us emphasize that for large values of  $N_t$  and  $K$ , BRB algorithm will not be applicable due to extraordinarily high complexity. However, the detailed computation complexity analysis of Algorithm 1 is omitted here due to the space limitation.

## V. CONCLUSION

We have presented a low computation complexity algorithm [see Algorithm 1] for finding a more accurate suboptimal solution for the WCUM problem in (6). The proposed algorithm has been proved to converge, and the limit point is Pareto optimal to the problem [see Proposition 1]. The presented simulation results have demonstrated that the proposed algorithm performs much better than SCA-based algorithm and has higher computational efficiency over both BRB and SCA-based algorithms.

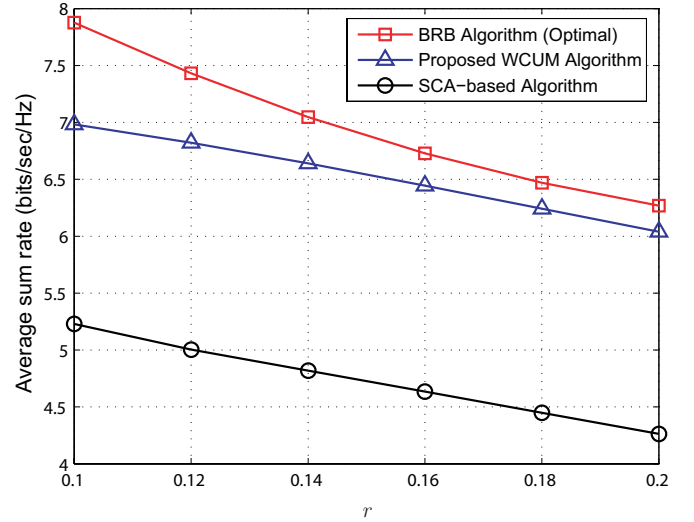


Fig. 2: Performance comparison of the proposed WCUM algorithm and two existing algorithms in terms of average sum rate versus CSI error radius  $r$ .

## ACKNOWLEDGMENTS

This material is based upon works supported by the National Science Council, R.O.C. under Grant NSC-99-2221-E-007-052-MY3, and by the National Science Foundation under Grants 1147930 and 0917251.

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